ACCELERATION OF A CONDUCTING GAS IN A STRONG NONSTATIONARY ELECTROMAGNETIC FIELD

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The problem of nonstationary acceleration of a conducting gas in a channel is solved, together with the problem of a discharge in an electric-circuit. As distinct from other papers, in which the typical solution assumed a thin cluster subject to acceleration, we examine the case in which the gas flow fills the entire channel. The motion of the gas in the channel is examined in one-dimensional formulation, under the assumption that the particle transit time in the channel is small compared to the discharge time and that the electromagnetic force is large compared to the pressure gradient.

For impulsive acceleration of the conducting gas, use is made of a discharge with a certain capacitance. Since the (time-variable) resistance of the channel and, consequently, the behavior of the discharge depend upon the channel flow of the conducting gas, the correct solution of the problem of gas acceleration in the induced electromagnetic field can be obtained only by analyzing simultaneously the magnetogasdynamic channel flow and the discharge process in the entire electric circuit. On the other hand, the acceleration of the gas itself is a function of the instantaneous potential difference at the electrodes. Hitherto, such simultaneous solutions were obtained by many investigators under the assumption, proposed in [1], that a channel gas flow may be treated as the motion of a unique narrow cluster, whose length is negligible as compared to the channel length. Experiments and theoretical estimates show, however, that in many cases the conducting gas fills the entire channel length during the acceleration process, so that the assumption of a narrow cluster is not even approximately fulfilled [4, 5].

1. Let us examine the nonstationary acceleration of a conducting gas in a channel of rectangular section, whose top and bottom walls are electrodes and whose side walls are insulators. We denote the channel length by l, the channel width by b, and the electrode gap by h. The voltage is applied to the electrodes from a capicitor bank with a total capacitance C and an initial charge Q_0, via an external circuit with a resistance R_0.

We consider our problem under the following assumptions:

a) external magnetic fields are absent, the gas is accelerated by the combined action of an electric field and magnetic self-field induced by the flowing currents. The gas conductivity is $\sigma = \text{const}$. Since the major portion of the current flows near the electrodes, the time-variation of the total circuit inductance may be neglected, i. e., it may be assumed that L = const;

b) the gas is accelerated to velocities from roughly $5 \cdot 10^6$ to 10^7 cm/sec, so that compared to the electromagnetic force, the pressure gradient may be neglected [2]. The initial velocity of the gas is small compared to the final velocity, and may be neglected in the computation of the magnetic field;

c) the electrodes are sufficiently long and wide to justify neglect of end effects (in particular, to assume a zero magnetic field strength (H = 0) at the channel outlet). The Hall currents are negligible. The gas is supplied uniformly across the channel inlet at a flow rate M(t);

d) the characteristic discharge time t_0 exceeds by far the particle transit time in the channel. Naturally, this assumption is not nearly always fulfilled. In view of this, we obtain an estimate which shows that this assumption holds in cases of interest in practice.

Let L = 60 cm, C = 1500 μF, l = 15 cm, and the outlet velocity $u_1 = 10^7$ cm/sec. Then $t_0 = 2\pi\sqrt{LC}/c \approx 6 \cdot 10^{-5}$ sec, while the transit time is on the order of $l/u_1 = 1.5 \cdot 10^{-6}$ sec. Thus, the ratio of the particle transit time in the channel to the characteristic discharge time is $2.5 \cdot 10^{-2}$.

Under these assumptions, the motion of the gas in the channel may be treated as one-dimensional. Furthermore, from this estimate it follows that in the determination of the relation between the voltage drop V at the electrodes and the total current I, the intantaneous velocity distribution may be taken as quasi stationary.

The system of equations from which the relationship between V and I can be determined has the form [2] (time is contained in these equations as a parameter, the x-axis is directed along the channel axis, the origin of the coordinates is located at the channel inlet section):

$$d(\rho u)/dx = 0, \quad \rho u du/dx = jH/c,$$
$$dH/dx = -4\pi j/c, \quad j = \sigma(V/h - uH/c). \tag{1.1}$$

where u is the velocity, ρ is the gas density, j is the electric current density, and c is the velocity of light in vacuo.

In the initial phase of motion, the gas does not extend over the entire channel length l but only over a certain length between x = 0 and $x = x_1$, where x_1 is the time-variable coordinate of the forward front. Hence, for $x = x_1$, the boundary condition H = 0 is fulfilled. After the forward front has reached the ends of the electrodes at a certain instant, the gas extends over the entire channel. From this time on, H = 0 may be assumed for $x = l$.

By integrating the third equation in (1.1) with respect to x from x = 0 to $x = x_1$ or to $x = l$, respectively, for the instant at which the gas extends over the entire channel and over only part of it, and integrating the first and second equations in (1.1) from zero to x, we obtain

$$\rho u = M/bh, \quad I = cbH_0/4\pi, \quad u = u_1(1 - H^{*2}), \tag{1.2}$$

$$H^* = \frac{H}{H_0}, \qquad u_1 = \frac{2\pi h I^2}{c^2 bM}, \qquad I = b\int_0^{x_1} j\,dx. \tag{1.3}$$

Here, H_0 is the magnetic field strength at the channel inlet section (for x = 0), and u_1 is either the velocity of the forward front of the gas before it has reached the ends of the electrodes, or the velocity at the end of the channel if the gas has extended over the entire channel at the instant under consideration.

By substituting (1.2) into the last equation in (1.1), allowing for (1.3), and integrating from zero to x_1 or to l, respectively, we obtain

$$R_1^* x_1^* = \int_0^1 \frac{dH^*}{\chi^2(H^{*3} - H^*) + 1}, \quad x_1^* = \frac{x_1}{l}, \quad R_1^* = \frac{\sigma lbV}{hI}, \quad \chi^2 = \frac{8\pi^2 h^2 I^3}{c^4 b^2 MV}. \tag{1.4}$$

Equation (1.4) yields the desired relationship between the potential difference V at the electrodes and the total current I. It should be combined with the discharge equations for the entire circuit, which can be written in the following form [3]:

$$dQ/dt = I, \quad (L/c^2)\,dI/dt + R_0 I + V + Q/C = 0. \tag{1.5}$$

In addition, for the initial phase of motion, it is necessary to include the equation of the forward front of the gas

$$dx_1/dt = u_1. \tag{1.6}$$

Equations (1.4)–(1.6) constitute a system of four equations for determining the variables Q, V, I, and x_1. The initial conditions for this system are

$$Q = Q_0, \quad V = 0, \quad I = 0, \quad x_1 = x_{10} \text{ for } t = 0. \tag{1.7}$$

Since relation (1.4) does not lend itself to solution with respect to the quantities required, system (1.4)–(1.6) must be transformed in order to determine the unknowns.

To this end, we introduce the unknown functions R_1^* and χ. From formulas (1.4), we have

$$I^2 = (c^4 b / 8\pi^2 \sigma lh)\, MR_1^* \chi^2, \quad V^2 = (c^4 h / 8\pi^2 \sigma^3 l^3 b)\, MR_1^{*3} \chi^2. \tag{1.8}$$

From here, dI/dt can be expressed in terms of the time-derivatives M, R_1^*, and χ. It should be noted that

$$x_1^* \frac{dR_1^*}{dt} = -R_1^* \frac{dx_1^*}{dt} - \frac{2}{\chi}\frac{d\chi}{dt}(R_1^* x_1^* - 2\psi), \qquad \psi = \frac{1}{2}\int_0^1 \frac{dH^*}{[\chi^2 (H^{*3} - H^*) + 1]^2}. \tag{1.9}$$

By substituting (1.3), (1.8), and (1.9) into (1.5) and (1.6) and introducing dimensionless variables, we get

$$\frac{d\chi}{d\tau} = -\frac{x_1^* \chi R_1^*}{4m\psi}\frac{dm}{d\tau} + \frac{\chi R_1^*}{4\psi}\frac{dx_1^*}{d\tau} - \frac{\beta x_1^* \chi R_1^* (R_0^* + R_1^*)}{2\psi} - \frac{2\pi^2 x_1^* q \sqrt{R_1^*}}{\alpha\psi\sqrt{m}},$$
$$dq / d\tau = \alpha\chi\sqrt{R_1^* m}, \qquad dx_1^* / d\tau = \gamma\chi^2 R_1^*, \tag{1.10}$$
$$q = Q / Q_0, \quad m = M / M_0, \quad \tau = t / t_0, \quad t_0 = 2\pi\sqrt{LC} / c,$$
$$\alpha = \left(\frac{c^2 b M_0 LC}{2\sigma l h Q_0^2}\right)^{1/2}, \quad \beta = \frac{2\pi c h}{\sigma b l}\left(\frac{C}{L}\right)^{1/2}, \quad \gamma = \frac{c\sqrt{LC}}{2\sigma l^2}, \quad R_0^* = R_0 \frac{\sigma b l}{h};$$

where M_0 is the characteristic value of the flow rate M. The boundary conditions (1.7) take the form

$$q = 1, \quad \chi = 0, \quad x_1^* = x_{10}^* \quad \text{for } \tau = 0. \tag{1.11}$$

System (1.4), (1.10) with the boundary conditions (1.11) lends itself readily to numerical integration.

2. In the special case in which the gas extends over the entire channel, while the time dependence of the flow rate roughly corresponds to an erosional supply of the working fluid, the solution of equations (1.4), (1.5) can be expressed in terms of elementary functions. In fact, M may be considered proportional to the total current I^2 or to the total power supplied to the gas, i. e., to VI, assuming that a definite portion of the total power is consumed for evaporation. If erosion takes place only at the channel inlet, the consumption is proportional to Joule heat release at the channel inlet section, i. e., to j_0^2. However, since $j_0 \sim V$ (the gas velocity is small at the inlet section, $uH \ll V/h$), in this case, we have $M \sim V^2$. In all these cases ($M = kI^2$, $M = kIV$, $M = kV^2$), the value of χ depends, to some degree, solely on the ratio V/I. In this case, however, it follows from formula (1.4) that for $x_1^* \equiv 1$, we have V/I = const. Hence, $V = R_1 I$, where R_1 is the constant resistance of the channel. By substituting this expression into (1.5) and writing the equations in dimensionless form, we obtain

$$dq / d\tau = i, \quad di / d\tau + \beta(R_0^* + R_1^*)i + 4\pi^2 q = 0, \tag{2.1}$$

where

$$i = (2\pi\sqrt{LC} / c Q_0) I.$$

Since system (2.1) has constant coefficients, its solution is expressed by well-known damped oscillation curves.

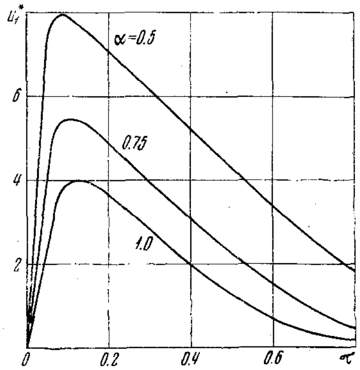

Fig. 1

3. **Figures** 1 and 2 show examples of computations from formulas (1.4), (1.10) for certain parameter values. Calculations were carried out for the case in which the conducting gas initially fills out the entire channel and the gas flow rate is constant in time. The values of the parameters were taken as: l = 15 cm, b = 5 cm, h = 3 cm, L = 60 cm, $\sigma = 5 \cdot 10^{12}$ sec^{-1}, $R_0 = 5 \cdot 10^{-4}$ ohm, C = 1500 μF. A constant gas flow rate was selected on the basis of the condition that a 0.001-g mass is accelerated during the time t_0 = 60 μsec (the channel dimensions and other parameters are taken

from conventional experimental conditions).

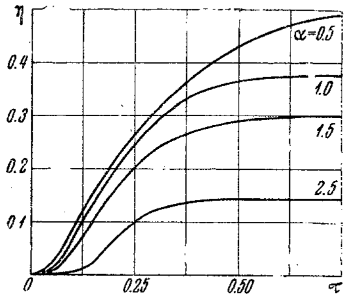

Fig. 2

Computations are performed for various values of parameter α, which correspond to various values of Q_0 (i. e., to various capacitor bank voltages). Figure 1 shows the dimensionless gas velocity $u_1^* = (4\pi \sigma l/c^2)u_1$ at the channel outlet section as a function of dimensionless time τ. From Fig. 1 it can be seen that, in agreement with general physical considerations, the gas velocity at the outlet is nonuniform when M = const. Rapidly increasing, it reaches a maximum at peak total current, and then declines again. In Fig. 2, the total kinetic energy of the gas is plotted against time for the same values of the parameters. The quantity

$$\eta = \left(\int_0^t \frac{Mu_1^2}{2} dt\right) \frac{2C}{Q_0^2}\,.$$

is plotted on the ordinate axis.

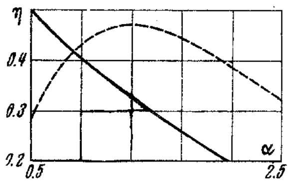

Fig. 3.

Of major interest is a comparison between the value of η obtained under the assumption that the gas extends over the entire channel and that a thin plasma cluster is accelerated [1]. In the latter case, we assume in the calculations that the cluster is accelerated in a channel of the same dimensions as in the case of continuous gas supply and the same capacitor bank and circuit inductance. The ratio dL/dx is assumed to be equal to the mean value of the inductance along the channel axis (for the channel dimensions considered, the mean value of $dL/dx \approx 4.5$), and the mass of the cluster to be equal to the total gas mass supplied (continuously) over the time t_0 (i. e., 0.001 g). The results of the calculations are shown in Fig. 3. The quantity η is plotted on the ordinate axis and the parameter α, which is inversely proportional to the initial voltage at the capacitor bank is plotted on the abscissa axis. The dashed curve represents data obtained under the assumptions employed in [1], while the solid curve represents data calculated in the present paper for the same parameter values as the curves in Fig. 2 and for $\tau = 1$. From Fig. 3, it can be seen that the results of calculations performed under various assumptions (continuous acceleration of the gas extended over the entire channel, and acceleration of a gas cluster) differ not only in the quantitative but also in the qualitative aspects of the relations $\eta(\alpha)$. In the case in which the gas fills the entire channel, the value of η is proportional to the initial voltage at the capacitor bank (inversely proportional to α). On the other hand, when only a gas cluster is accelerated, η has a maximum at a certain value of α, since for a high initial voltage at the capacitor bank, the cluster is rapidly accelerated to a high velocity and is expelled from the channel during the initial phase of the discharge, the larger portion of the energy stored in the capacitors not being utilized.

From the data in Fig. 3 it follows that when the gas fills the entire channel, the formulas derived for the acceleration of a narrow gas cluster are not applicable for calculating the flow parameters (in particular η), as has been done in several papers. The influence of the characteristics of a channel flow of a conducting gas on the discharge process is shown in Fig. 4. In this figure the dimensionless time τ is plotted on the ordinate axis, and the dimensionless current i in the circuit is plotted with inverse sign on the abscissa axis. The calculations are performed for a channel with l = 15 cm, h = 3 cm, b = 5 cm, a capacitor bank with C = 1500 μF, Q_0 = 2.46 coul, an external circuit resistance $R_0 = 5 \cdot 10^{-4}$ ohm, L = 60 cm, and $\sigma = 5 \cdot 10^{12}$ sec^{-1}. The solid curve in Fig. 4 shows the discharge current as a function of time for a constant mass flow rate ($\dot{M}$ = const), and the dashed line for $\dot{M} = kI^2$, for $k \approx 10^{-27}$ sec^3/cm^3. A mass of 0.001 g is accelerated during a time $t_0 = 2\pi\sqrt{LC}/c \approx 60$ μsec in both cases.

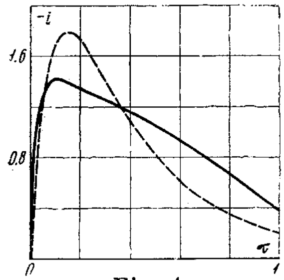

Fig. 4

From Fig. 4, it can be seen that the conditions of working fluid supply have a substantial influence on the discharge behavior. At a constant gas flow rate, the velocity for τ close to zero and τ close to unity is appreciably smaller (see Fig. 1) than at peak current, since a smaller force acts on the same mass. This is the reason why the back emf (the quantity uH/c) is also appreciably smaller, while at the same time the current is larger than in the case of gas supply according to the law $\dot{M} = kI^2$, where the characteristic value of the velocity in the channel is constant during the entire duration of the discharge.

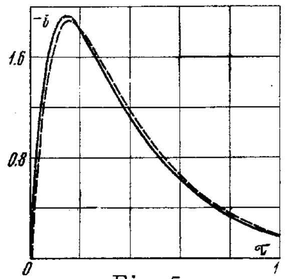

Fig. 5

When the gas does not extend over the entire channel length, the initial period has a certain influence on the formation of the discharge process. Figure 5 shows the behavior of the current with time for the test parameters previously employed, for $\dot{M} = kI^2$, but for a channel with l = 30 cm, and Q_0 = 4.5 coul. The solid curve is plotted without allowance for the initial period, i. e., under the assumption that the conducting gas immediately expands over the entire channel. The dashed curve is plotted with allowance for a gradual expansion of the gas over the entire channel length. From Fig. 5, it can be seen that a noticeable difference in the current can be observed only during the initial period, lasting slightly longer than the time required to fill the channel with gas (in the case under consideration, the forward front of the gas needs τ = 0.1 to reach the end of the channel). The proximity of the curves in Fig. 5 confirms the assumption that, in the calculations the channel flow may be treated as quasi stationary, provided the particle transit time in the channel is much smaller than the characteristic discharge time.

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